

Roll No.....

Total No. of Page(s): 2

BT-3/ D-21: 43135
BS-205 A : Mathematics-III

[Max. Marks: 75]

Time: 3 Hours]

Note: Attempt any five questions.

1 (a) Test the convergence of $\sum_{n=1}^{\infty} \left(\frac{(n+1)(n+2)}{n^2 \sqrt{n}} \right)$

(b) Discuss the convergence or divergence of the series

$$\frac{x}{1.2} + \frac{x^2}{2.3} + \frac{x^3}{3.4} + \dots \quad x > 0$$

2 (a) Expand $f(x) = x \sin x$ as a Fourier series in $(0, 2\pi)$.

(b) Find the half-range sine series for $f(x) = x(\pi - x)$ in the interval $(0, \pi)$ and deduce that

$$\frac{1}{1^3} - \frac{1}{3^3} + \frac{1}{5^3} - \frac{1}{7^3} + \dots = \frac{\pi}{32}$$

3 (a) Solve $(x^2y - 2xy^2)dx - (x^3 - 3x^2y)dy = 0$ using exact differential equation.

(b) Solve the differential equation

$$x^2 \left(\frac{dy}{dx} \right)^2 + xy \frac{dy}{dx} - 6y^2 = 0$$

4 (a) Solve $\frac{d^3y}{dx^3} - 6\frac{d^2y}{dx^2} + 11\frac{dy}{dx} - 6y = e^{2x}$

(b) Solve by the method of variation of parameters:

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} = e^x \sin x$$

5 (a) Change the order of integration in the interval:

$$\int_0^{4a} \int_{\frac{x^2}{4a}}^{2(ax)^{\frac{1}{2}}} dy dx$$

(b) Show that area between the parabolas $y^2 = 4ax$ and $x^2 = 4ay$ is

$$\frac{16}{3} a^2$$

6 (a) Evaluate

$$\int_1^e \int_1^y \int_1^x \log z \, dz \, dx \, dy$$

(b) Find the volume of the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$

7 (a) If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, show that $\Delta r^n = nr^{n-2}\vec{r}$

(b) Prove that $\nabla^2 f(r) = f''(r) + \frac{2}{r}f'(r)$

8 (a) Evaluate the line integral $\int_C (x^2 + xy) \, dx + (x^2 + y^2) \, dy$,

where C is the square formed by the lines $x = \pm 1, y = \pm 1$

(b) Verify Green's Theorem for $\oint_C [(xy + y^2) \, dx + x^2 \, dy]$,

where c is bounded by $y = x$ and $y = x^2$.